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SOME PROBLEMS OF MAGNETOSTATIC AMPLIFIER THEORY

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1. The first part of this paper deals with the problem of exciting oscillations in a small gyrotropic sphere placed in a waveguide and the scattered waves that arise as a result. This problem is of interest from the standpoint of measuring the parameters of ferrite samples, which are usually carried out in waveguides and resonators, and accounting for the effect of the ferrite samples on the natural frequencies of the resonators. Our investigation is based on Walker's solution to the problem of determining the magnetic modes of a small gyrotropic spheroid /1/. Experiments show Walker's results to be in quite good agreement with ferromagnetic resonance phenomena, which are observed in the microwave band.

Walker started out from the Landau-Lifshitz equation and also the following two magnetic field equations:

$$\begin{aligned} \mathrm{Curl}\,\vec{h} &= 0 \\ \mathrm{div}\,(\vec{h} + 4\pi\vec{m}) &= 0 \end{aligned} \qquad (1)$$

This enabled him to come to the following magnetic potentials by putting $\vec{h} = \mathrm{grad}\,\psi$:

internal magnetic potentials:

$$\psi_{in}^{\pm m} \sim P_n^m(i\xi')\,P_n^m(\eta')\,e^{\pm im\varphi}$$

external magnetic potentials:

$$\psi_{en}^{\pm m} \sim Q_n^m(i\xi)\,P_n^m(\eta)\,e^{\pm im\varphi}$$

Here P_n^m and Q_n^m are associated Legendre polinomials of the first and second order;

ξ', η' and φ are a set of spheroidal coordinates inside the spheroid;

ξ, η and φ are the same, but outside the spheroid.

The boundary conditions for ψ at the spheroid surface require continuity of the potential ψ and of the normal component of the induction $\vec{b} = \vec{h} + 4\pi\vec{m}$, This leads to a set of two homogeneous equations:

$$\psi_i = \psi_e \qquad (3)$$
$$b_{ni} = b_{ne}$$

By equating the determinant of this set to zero, Walker obtained the characteristic equation for determining the resonant frequencies or the resonant values of the d.c. magnetic field, at which oscillations arise without any action on the part of the external field (if losses are neglected, of course):

$$\mathcal{F} = m\alpha^2(\pm V) + i\xi_0' \frac{P_n'^m(i\xi_0')}{P_n^m(i\xi_0')} - i\xi_0 \frac{Q_n'^m(i\xi_0)}{Q_n^m(i\xi_0)} = 0 \qquad (4)$$

The notation here is the same as in Walker's paper, that is:

$\alpha = \dfrac{b}{a}$ is the ratio of the spheroid axes (fig.1);

ξ_0 and ξ_0' are the coordinates of the spheroid surface in the external and internalcoordinate sets;

$$V = \frac{-\Omega}{\Omega_H^2 - \Omega} \qquad \text{where,} \quad \Omega = \frac{\omega}{4\pi/\chi/M_0} \qquad \Omega_H = \frac{H_c}{4\pi M_0}$$

Figure 5 shows the relationship between the d.c. magnetic field H_i and the resonant frequencies of the gyrotropic sphere obtained by solving equation (4).

In contradistinction to Walker we must introduce into equation (3) the electromagnetic field of a wave moving along a waveguide, which causes the sphere to oscillate.

It is convenient for us to express the electromagnetic field components by means of the Debye potentials U_i and V_i, which correspond to transverse-magnetic and transverse-electric spherical waves:

$$E_{ri} = \frac{\partial^2 (rU_i)}{\partial r^2} + K_0^2 r U_i$$

$$E_{\theta i} = \frac{1}{r} \frac{\partial^2 (rU_i)}{\partial r \partial \theta} - \frac{iK_0}{r \sin\theta} \frac{\partial (rV_i)}{\partial \varphi}$$

$$E_{\varphi i} = \frac{1}{r \sin\theta} \frac{\partial^2 (rU_i)}{\partial r \partial \varphi} + \frac{iK_0}{r} \frac{\partial (rV_i)}{\partial \theta}$$

$$h_{ri} = \frac{\partial^2 (rV_i)}{\partial r^2} + K_0^2 r V_i$$

$$h_{\theta i} = \frac{1}{r} \frac{\partial^2 (rV_i)}{\partial r \partial \theta} + \frac{iK_0}{r \sin\theta} \frac{\partial (rU_i)}{\partial \varphi}$$

$$h_{\varphi i} = \frac{1}{r \sin\theta} \frac{\partial^2 (rV_i)}{\partial r \partial \varphi} - \frac{iK_0}{r} \frac{\partial (rU_i)}{\partial \theta}$$

The field inside the waveguide may be represented by two series of potentials U_{in}^{m} and V_{in}^{m}, which are of degree n and order m; for small values of $K_0 r$ we have:

$$V_{in}^{\pm m} = iU_{in}^{\pm m} \sim j_n (K_0 r) P_n^m (\cos\theta) e^{\pm im\varphi} \qquad (6)$$

By considering the field in the waveguide to be the sum of plane waves and by resolving the plane waves into spherical waves, we can obtain the following series for the Debye potentials of the field at various points inside the waveguide:

$$U_i = \sum_{n,m} a_n^m U_{in}^m \;; \qquad V_i = \sum_{n,m} b_n^m V_{in}^m .$$

Figure 2 gives curves for the coefficients $a_1^{-1}\, b_1^{-1}\, a_2^{-1}\, b_2^{-1}$ etc. of these series for circularly polarized normal waves as a function of the distance of the selected point from the waveguide wall. Circular polarization explains the unsymmetrical shape of the curves and the existence of zero values for the coefficients. Straight lines in the figure correspond to coefficients a_n^{-1} and b_n^{-1} for the plane wave moving in the $-z$-direction with the magnetic vector parallel to the x-axis.

Before introducing the exciting field into the boundary conditions (5), we must determine the relationship between the Debye potential V and the Walker or magnetostatic potential Ψ. It is not difficult to show that the following relationship is valid for small values of $k_0 r$:

$$\frac{\partial}{\partial r}\left(r V_n^{\pm m}\right) \approx \Psi_n^{\pm m} \qquad (7)$$

Making use of this relationship and the boundary conditions, we obtain for small sphere radii a the following expressions for the amplitude $A_n^{\pm m}$ of the potential at the surface of the sphere and for the amplitude $C_n^{\pm m}$ of the Debye potential of the scattered wave (corresponding to the exciting potential $V_n^{\pm m}$):

$$A_n^{\pm m} = \frac{(K_0 a)^n}{1\cdot 3\cdot 5\cdots(2n-1)} \frac{1+n}{1+n+q_\pm}$$

$$C_n^{\pm m} = \frac{(K_0 a)^{2n+1}}{[1\cdot 3\cdot 5\cdots(2n-1)]^2} \frac{1+n}{n(2n+1)} \left(\frac{2n+1}{1+n+q_\pm} - 1\right) \tag{8}$$

The following approximate formulae were used for the Bessel and Hankel spherical functions $j_n(K_0 r)$ and $h_n(K_0 r)$ in deriving these expressions:

$$j_n(K_0 r) \approx \frac{(K_0 r)^n}{1\cdot 3\cdot 5\cdots(2n+1)} \tag{9a}$$

$$h_n^{(2)}(K_0 r) \approx \frac{i\, 1\cdot 3\cdot 5\cdots(2n-1)}{(K_0 r)^{n+1}} \tag{9b}$$

In equation (8)

$$q_\pm = \pm m V + \sqrt{\frac{1+K}{K}} \; \frac{P_n^{\prime m}\left(\sqrt{\frac{1+K}{K}}\right)}{P_n^m\left(\sqrt{\frac{1+K}{K}}\right)} \tag{10}$$

where, $K = \dfrac{\Omega_H}{\Omega_H^2 - \Omega^2}$ as is with Walker.

Note that the denominator $1+n+q_\pm$ is the left-hand side of the characteristic equation for the sphere

$$1+n+q_{\pm}=0 \tag{11}$$

obtained from equation (4) by putting $\alpha=1$.

From equation (8) we see that in the case of small spheres the amplitudes of the oscillations of higher orders (n) decrease rapidly with the order number since they are proportional to ($K_0 a$)^n . The amplitudes of the scattered wave potentials decrease even more rapidly as they are proportional to $(K_0 a)^{2n+1}$.

By introducing losses, we can calculate the amplitude of the oscillations for resonance conditions. Relative values of the resonance amplitudes for three Walker modes ($1,-1,0$), ($2,-1,0$) and ($3,-1,0$) with $K_0 a=1$ are given in figure 3.

Our investigation showed that the width of the resonance curve obtained by varying the d.c. field at a constant frequency remains the same for all modes.

Hitherto we have found the transverse-electric component of the scattered spherical wave. The second component (transverse-magnetic in nature) is produced in our case by the electric field of the sphere. It is evaluated from the following two equations:

$$\mathrm{Curl}\,\vec{E}=-\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \tag{12}$$

$$\mathrm{div}\,\vec{E}=0$$

plus the Debye potentials $U_{i\ell}^{m}$ of the waveguide field. A detailed investigation shows, however, that due to the gyrotropic properties of the sphere, the above mentioned oscillations of the TE type may produce additional potentials U_n^m of the TM type. For instance, oscillations of the type V_2^1 may produce the Debye potential U_1^1 at the surface of the ferrite sphere. However, the latter potentials are usually small as compared with the potentials of the waveguide field except when $U_{im}^m = 0$, or the ferrite losses are very small ($\Delta H \leq 1$ oersted).

By solving the problem of a dielectric sphere excited by the Debye potential $U_n^{\pm m}$, we obtain the following expression for the amplitude of this potential in the scattered TM wave:

$$B_n^{\pm m} = \pm \frac{1}{2} \frac{(K_0 a)^{2n+1}}{[1 \cdot 3 \cdots (2n-1)]^2} \times \frac{(n+1)}{2n+1} \frac{\varepsilon - 1}{n\varepsilon + n + 1} \tag{13}$$

For resonance conditions this amplitude is much smaller than the above mentioned amplitude C_n^m. In the latter case it is sufficient to consider only the TE scattered wave of amplitude C_n^m.

The scattered spherical waves hitherto investigated occur in free space. In the waveguide they are transformed into normal waveguide waves. Moreover, when evaluating the amplitude of any normal mode (for instance H_{10} in a rectangular waveguide), we must take into account spherical waves having all possible values for the indices n and m.

The problem of transforming spherical waves into normal waveguide waves was solved using the Lorentz lemma /2/ and the Debye potentials for normal waves in the waveguide.

Here are two examples of the results we obtained in this way.

When the d.c. magnetic field is parallel to the short wall of the waveguide, the backward reflection coefficient $\tau_{\beta n}^{m}$ of the H_{10} wave is as follows:

for mode (1, - 1,0) oscillations:

$$\tau_{\beta_1}^{-1} = \frac{-2\pi i (K_0 a)^3}{K_y a' b K_0^3} (K_y^2 \sin^2 K_x d - K_x^2 \cos^2 K_x d) \left[\frac{3}{3+K-V} - 1\right]. \quad (14a)$$

for mode (2, - 2,0) oscillations:

$$\tau_{\beta_2}^{-2} = \frac{-\pi i (K_0 a)^5}{6 K_y a' b K_0^3} \Big[(K_y \cos\beta_1 - K_x \sin\beta_1)^2 \cos^2 K_x d - \quad (14b)$$

$$- (K_y \sin\beta_1 + K_x \cos\beta_1)^2 \sin^2 K_x d\Big] \left[\frac{5}{5+2(K-V)} - 1\right]$$

The corresponding forward reflection coefficients $\tau_{f_1}^{-1}$ and $\tau_{f_2}^{-2}$ for the field of the H_{10} wave created by the sphere in the direction of wave travel are as follows:

$$\tau_{f_1}^{-1} = \frac{2\pi i (K_0 a)^3}{K_y a' b K_0^3} (K_y \sin K_x d + K_x \cos K_x d)^2 \left[\frac{3}{3+K-V} - 1\right], \quad (14c)$$

$$\tau_{f_2}^{-2} = \frac{\pi i (K_0 a)^5}{6 h_y a' b K_0^3} \Big[(K_y \cos\beta_1 - K_x \sin\beta_1) \cos K_x d -$$

$$- (K_y \sin\beta_1 + K_x \cos\beta_1) \sin K_x d\Big] \left[\frac{5}{5+2(K-V)} - 1\right] \quad (14d)$$

Here d is the distance from the center of the sphere to the short wall of the waveguide (fig. 4);

K_x and K_y are the propagation constants of the H_{10} wave in the x and y directions and

$$\beta = \arctan\left(-\frac{K_y}{K_x}\right)$$

a' and b are the dimensions of the short and long walls of the waveguide.

Fig. 4. gives calculated curves of the forward and backward reflection coefficients for $K_x = K_y$ as a function of the position of the sphere in the waveguide.

Note that for the mode ($1, -1, 0$) a small sphere placed at $K_x d = 45^0$ has a scattered wave only in the direction along which the normal wave travels. Experimental values of forward reflection coefficients are denoted by dots on fig. 4.

2. The second part of this paper analyzes the oscillations of a small gyrotropic spheroid when subjected to a large pumping field of frequency ω_3, which sets up a magnetization m_3 in the sphere. Due to the nonlinearity of the Landau-Lifshitz equation oscillations of two frequencies ω_1 and ω_2 must exist at the same time, such that

$$\omega_1 + \omega_2 = \omega_3 \qquad (15)$$

This problem was solved by expanding the magnetostatic potentials φ_1 and φ_2 in a power series about small parameter δ_0 :

$$\begin{aligned} &\omega_1 + \omega_2 = \omega_3 \\ &\varphi_1 = \varphi_1' + \delta_0 \varphi_1'' + \delta_0^2 \varphi_1''' + \cdots\cdots \\ &\varphi_2 = \varphi_2' + \delta_0 \varphi_2'' + \delta_0^2 \varphi_2''' + \cdots\cdots \end{aligned} \qquad (16)$$

where: $\delta_0 = \dfrac{m_3}{M} (K_1 - V_1 + K_2 - V_2)$

It turns out that the additional magnetic potential of coupled oscillations at frequency ω_1 (or ω_2) arising from the first order term of pumping magnetization δ_0 may be represented as the sum of all magnetostatic modes existing at frequencies ω_2 (or ω_1):

$$\psi_1 = \sum_{n,m,l} \psi'_{1\,n,m,l} + \delta_0 \sum_{n,m,l} A_{2\,n,m,l}\, \psi'_{2\,n,m,l}$$

$$\psi_2 = \sum_{n,m,l} \psi'_{2\,n,m,l} + \delta_0 \sum_{n,m,l} A_{1\,n,m,l}\, \psi'_{1\,n,m,l}$$

(17)

A method was derived for calculating the power series coefficients A_1 and A_2.

The boundary conditions requiring continuity of potentials ψ_1 and ψ_2, and also of the normal components of the induction B_1 and B_2 at the spheroid surface result in a set of four homogeneous equations with respect to the amplitudes of the oscillations and scattered waves. Putting the determinant of this set equal to zero, we obtain:

$$F_1 \cdot F_2 - \left(\frac{m_3}{M}\right)^2 F_3\, F_4 = 0 \qquad (18)$$

where F_1 and F_2 are the left-hand sides of Walker's transcendental equation (4) for oscillations of frequency ω_1 and ω_2 , and

$$F_3 = \alpha^2 \sqrt{1+K_2}\left(\frac{1-\alpha^2(1+K_1)}{1-\alpha^2(1+K_2)}\right)^{\frac{1}{2}(m_1+2)} \frac{4\,(2m_1+3)}{2\alpha^2 m_2 (1+K_1)+1}\left\{\left[(K_1-V_1)+(K_2-V_2)\right]\times\right.$$

$$\left.\times\left[\frac{2\alpha^2 m_2 (1+K_1)}{2\alpha^2 m_2 (1+K_1)+1} - \frac{2\alpha^2 m_2 (1+K_2)}{2(2m_1+3)\alpha^2 (K_1-K_2)} F_1\right] + (K_1-V_1)\right\}$$

$$F_4 = \frac{\alpha^2}{\sqrt{1+K_2}\;\, 2(2m_2+1)}\left(\frac{1-\alpha^2(1+K_2)}{1-\alpha^2(1+K_2}\right)^{\frac{1}{2}(m_2+1)}\left[2m_2(K_2-V_2) - \frac{1}{\alpha^2(1+K_1)}(K_1-V_1)\right]$$

From here we can determine the relationship between shifts of the resonant frequency $\delta\omega_1$, (for the given mode) and of the resonant d-c. magnetic field δH in the presence of a pumping field. The following equation turns out to be valid for small shifts.

$$\left(\frac{\delta\omega_1}{\gamma}\right)^2 = (\delta H)^2 - \left(\frac{m_0}{2}\right)^2 \frac{F_3 \, F_4}{\frac{\partial F_1}{\partial H} \, \frac{\partial F_2}{\partial H}} \qquad (18a)$$

$$F_1 = 0 \qquad F_2 = 0$$

The second term in the right-hand side of this equation is positive for amplification conditions. Under these conditions, the frequency shift $\delta\omega_1$ should be imaginary. It follows, therefore, that it is most expedient to retain the previous value for the resonant field ($\delta H = 0$). If we set $i\frac{\delta\omega_1}{|\gamma|} = \Delta H$, we may find the threshold level for the pumping power from equation (18a). However, we determine this level by other means further on.

If no pumping field is present ($\delta_0 = 0$) equation (18) may be decomposed into the two Walker equations mentioned above:

$$F_1 = 0 \qquad F_2 = 0$$

A very important rule was derived on the basis of analysis concerning the modes at the frequencies ω_1 and ω_2, which are capable of interacting with each other, that is, which are parametrically coupled. This rule states that only the following pair of modes will arise in a ferrite ellipsoid when subjected to a strong pumping field: $(n_1, n-2, 1)$ and $(n, 1-n, 0)$.

This rule considerably facilitates analysis of the performance of a magnetostatic amplifier and in general of nonlinear phenomena in ferrite.

In order to carry out further investigations, it was necessary to account for energy losses and exciting potentials, just as was the case in the first part of this paper. After making several transformations we arrive at the following two equations containing two unknowns, the oscillation amplitudes a_1 and a_2:

$$a_1(G_1 \Delta H_1 - iF_1) - i\frac{m_0}{2}a_2^* F_3 = i\tau_1 V_1$$
$$a_2(G_2 \Delta H_2 - iF_2) - i\frac{m_0}{2}a_1^* F_4 = i\tau_2 V_2 \qquad (19)$$

Here τ_1 and τ_2 are the amplitudes of the exciting potentials,

$$G_{1,2} = \frac{\partial F_{1,2}}{\partial H} + F_{1,2}\frac{P'^{m_{1,2}}_{n_{1,2}}(j\xi'_{0\,1,2})}{P^{m_{1,2}}_{n_{1,2}}(j\xi'_{0_{1,2}})}\, j\, \frac{\partial \xi'_{0\,1,2}}{\partial H}$$

$$V_{1,2} = j\xi_0\left[\frac{Q'^{m_{1,2}}_{n_{1,2}}(j\xi_0)}{Q^{m_{1,2}}_{n_{1,2}}(j\xi_0)} - \frac{P'^{m_{1,2}}_{n_{1,2}}(j\xi_0)}{P'^{m_{1,2}}_{n_{1,2}}(j\xi_0)}\right]\frac{P^{m_{1,2}}_{n_{1,2}}(j\xi_0)}{P^{m_{1,2}}_{n_{1,2}}(j\xi'_{0\,1,2})}$$

A similar equation for pumping field oscillations which account for the effect of magnetostatic coupled oscillations, may be added to equations (19):

$$h_3 = \frac{m_0}{2}\left[\Delta H_3 + i\,(H_{03} - H)\right] + a_1 a_2 q$$

where

$$q = \frac{1}{4\pi M}\;\frac{\int_V h_{z_1} 4\pi(m_{x_2} + jm_{y_2})dV + \int_V h_{z_2} 4\pi(m_{x_1} + jm_{y_1})dV}{V_f} \qquad (20)$$

In the above $h_{z_{1,2}}$ and $m_{x,y_{1,2}}$ denote the longitudinal magnetic field and the magnetic moment of coupled modes at the frequency ω_1 and ω_2, respectively, and V_f represents the volume of the ferrite ellipsoid. Integration is carried out over the entire volume of the ferrite.

The set of equations in (19) is similar from the mathematical standpoint to the current equations for parametrically coupled circuits in the form given by Bloom and Chang /3/. For example, the oscillation amplitudes $a_{1,2}$ correspond to the currents in the coupled circuits $I_{1,2}$; the quantity $\frac{m_0}{2}$ corresponds to the pumping current I_3 , etc. By making use of this analogy, we can very simply obtain several important formulas for ferrite subjected to a strong pumping field, which are similar to formulas for parametrically coupled circuits. These formulas are listed in the table. They enable the main characteristics of a ferrite spheroid subjected to a strong pumping field to be calculated. This spheroid can be imagined as an oscillating system with many degrees of freedom, which account for all oscillations whose indices comply with the above mentioned relationship.

In particular, these expressions can be used to determine the values of the d.c. magneting field, which correspond to conditions of subsidiary absorption as described by Bloembergen and Wang /4/. For values of the d.c. magnetizing field satisfying equation (5T) x), maximum attenuation is introduced into the pumping field oscillations, which means that a noticeable amount of subsidiary absorption appears. The condition for maximum subsidiary absorption requires that the pumping amplitude be close to the critical value as is seen from formula (4T). From equation (5T) we can determine the bounds of the subsidiary absorption region for a given pumping amplitude. The lines in fig. 4 correspond to $F_1 = F_2 = 0$

for some Walker modes. The vertical line, which intersects the lines for modes (2,0,1) and (2,-1,0) at points whose ordinates are f_1 and f_2 and give the pumping frequency f_3 as their sum, demarks the subsidiary absorption field on the abscissa. To the right is demarked the field corresponding to resonance conditions of uniform procession (1,-1,0) for a pumping frequency f_3.

Equation (5r) may be satisfied, however, with non-zero values of F_1 and F_2. This will be the case at other values of the frequencies f_1 and f_2, which nevertheless add up to f_3. By varying f_1 from 0 to f_3, we can obtain the region for subsidiary absorption created by modes (2,0,1) and (2,-1,0) (fig. 5). This region is the first to appear when gradually increasing the pumping power. For a further strengthening of the pumping field, other pairs of modes begin to interact and the subsidiary absorption region expands.

Fig. 6 gives curves of the critical magnetic pumping field for $\Delta H = 1$ oe with the earlier mentioned modes (2,0,1) and (2,-1,0). Curves are calculated for different values of saturation magnetisation Mo as a function of the spheroid parameter $\alpha = \frac{b}{a}$ with $F_1 = F_2 = 0$. They have a shallow minimum at $\alpha \sim 1,2$, that is, when the spheroid is close to the sphere in shape. Knowing the parameters of the ferrite and the resonator and using these curves, we can calculate the power of the pumping field which is critical for the above two modes. The lowest value of this critical power for these modes occurs, however, at certain non-zero values of F_1 and F_2. The corresponding d.c. magnetizing fields lie in the shaded zone of fig. 5. The results of calculations carried out on the basis of the above theory compare well with experiment.

References

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4. N.Bloembergen, S.Wang, - Phys. Rev., 1954, 93, 72.

Table I

Parametrically coupled circuits	Ferrite when acted upon by a large pumping field
1. Resistance induced in the circuit of frequency ω_1	1. Change in the width of the ferromagnetic resonance curve at frequency ω_1
$R_{\theta H_1} = -\|J_3\|^2 \frac{2\omega_1 \omega_2 \mathcal{L}^2}{R_2^2 + X^2} R_2$	$\delta(\Delta H)_1 = -\left(\frac{m_0}{2}\right)^2 \frac{F_3 F_4}{(G_2 \Delta H_2)^2 + F_2^2} \frac{G_2 \Delta H_2}{G_1}$
2. Power gain at resonance	2. Loss compensation coefficient for coupled oscillations in the case of ferromagnetic resonance
$y_0^{\frac{1}{2}} = \left(1 - \frac{\|J_3\|^2}{\|J_3\|^2_{\kappa p}}\right)^{-\frac{1}{2}}$	$y_0^{\frac{1}{2}} = \left[1 - \frac{\left(\frac{m_0}{2}\right)^2}{\left(\frac{m_0}{2}\right)^2_{\kappa p}}\right]^{-\frac{1}{2}}$
3. Frequency bandwidth equation for $y = \frac{1}{2} y_0$	3. Equation for computing the frequency bandwidth and the magnetic field for $y = \frac{1}{2} y_0$
$\frac{X_1}{R_1} - \frac{\|J_3\|^2}{\|J_3\|^2_{\kappa p}} \cdot \frac{X_2}{R_2} = 1 - \frac{\|J_3\|^2}{\|J_3\|^2_{\kappa p}}$	$\frac{F_1}{G_1 \Delta H_1} - \frac{(m_0/2)^2}{(m_0/2)^2_{\kappa p}} \frac{F_2}{G_2 \Delta H_2} = 1 - \frac{(m_0/2)^2}{(m_0/2)^2_{\kappa p}}$

Parametrically coupled circuits	Ferrite when acted upon by a large pumping field
4. Modulus of the current in circuit 1.	4. Amplitude of coupled magnetostatic oscillations
$J_1 = \dfrac{E_1/R_1}{\sqrt{\left[1-\dfrac{\lvert J_3\rvert^2}{\lvert J_3\rvert^2_{\kappa p}}\right]^2+\left[\dfrac{X_1}{R_1}-\dfrac{\lvert J_3\rvert^2}{\lvert J_3\rvert^2_{\kappa p}}\dfrac{X_2}{R_2}\right]^2}}$	$a_1 = \dfrac{\tau_1 V_1/G_1\Delta H_1}{\sqrt{\left[1-\dfrac{(m_0/2)^2}{(m_0/2)^2_{\kappa p}}\right]^2+\left[\dfrac{F_1}{G_1\Delta H_1}-\dfrac{(m_0/2)^2}{(m_0/2)^2_{\kappa p}}\dfrac{F_2}{G_2\Delta H_2}\right]^2}}$
5. Equations for computing the frequencies corresponding to a maximum resistance induced in circuit 3 and the generated frequencies for $J_3 = J_{3\,\kappa p}$.	5. Equations for computing the d.c. magnetic field intensity corresponding to the subsidiary absorption and the generated frequencies for $\dfrac{m_0}{2} = \left(\dfrac{m_0}{2}\right)_{\kappa p}$
$\dfrac{X_1}{R_1} - \dfrac{X_2}{R_2} = 0$	$\dfrac{F_1}{G_1\Delta H_1} - \dfrac{F_2}{G_2\Delta H_2} = 0$
6. Threshold level	6. Condition of total loss compensation for coupled magnetostatic oscillations
$\lvert J_3\rvert^2_{\kappa p} = \dfrac{R_1(R_2^2+X_2^2)^2}{\omega_1\omega_2\mathcal{L}\cdot R_2}$	$\left(\dfrac{m_0}{2}\right)^2_{\kappa p} = \dfrac{\Delta H_1\Delta H_2}{\omega_m^2}\,\varkappa^{-2}\left[1+\left(\dfrac{F_{1,2}}{G_{1,2}\Delta H_2}\right)^2\right],$ где $\varkappa = \left(\dfrac{F_3 F_4}{G_1 G_2\,\omega_m^2}\right)^{\frac{1}{2}}$

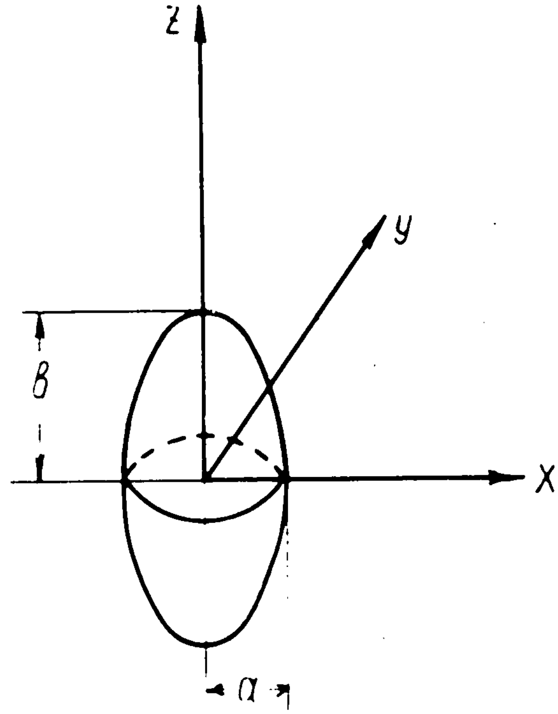
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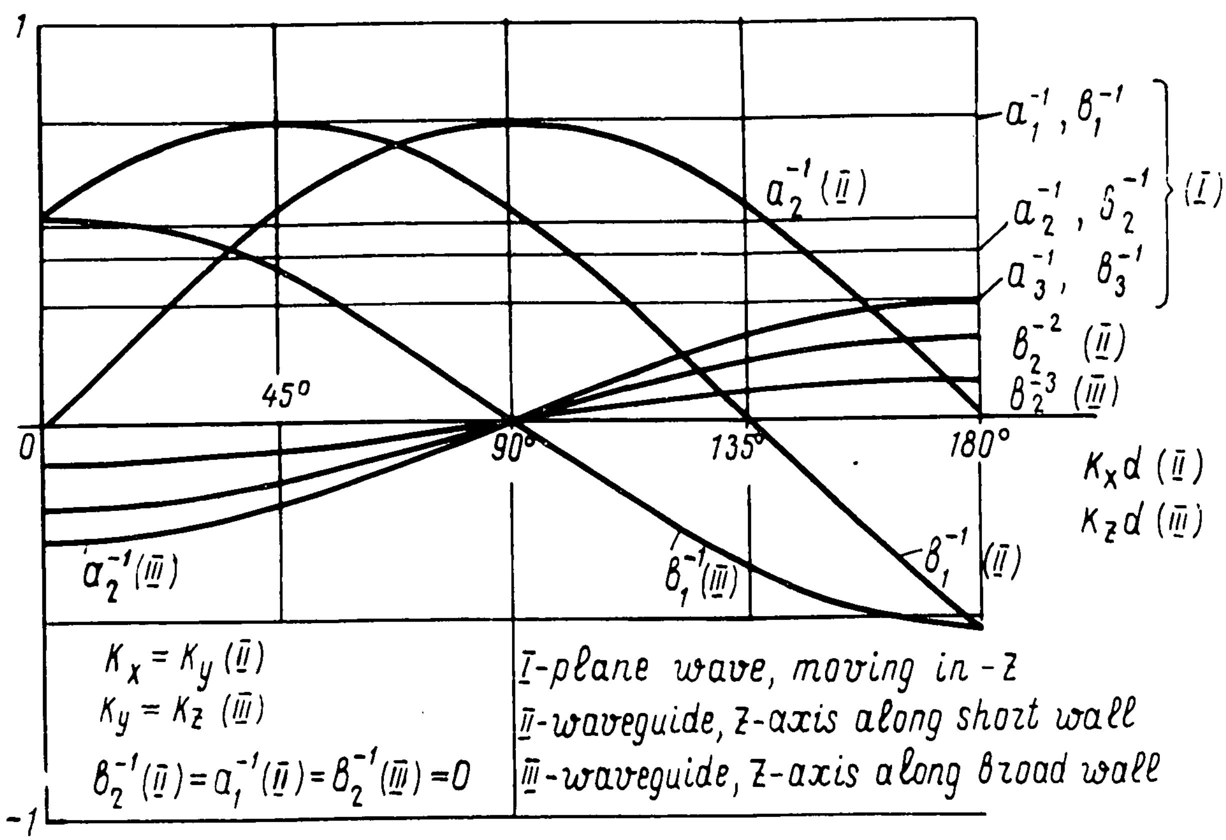
Fig. 1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a_1^{-1}, b_1^{-1}
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(I)
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$K_x = K_y$ (II)
$K_y = K_z$ (III)
$b_2^{-1}(II) = a_1^{-1}(II) = b_2^{-1}(III) = 0$
I-plane wave, moving in -z
II-waveguide, z-axis along short wall
III-waveguide, z-axis along broad wall

Fig. 2.

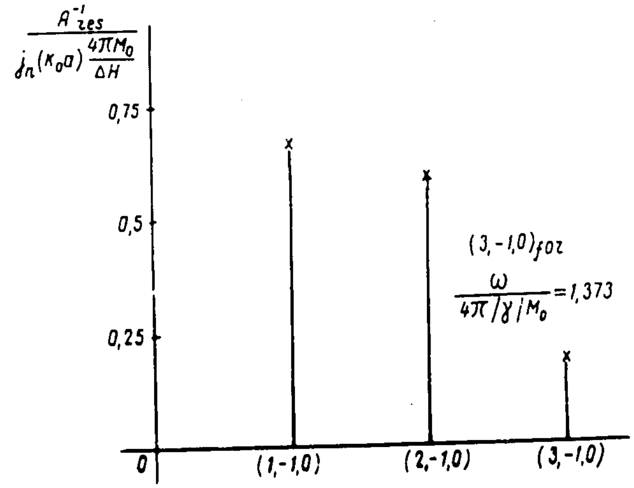
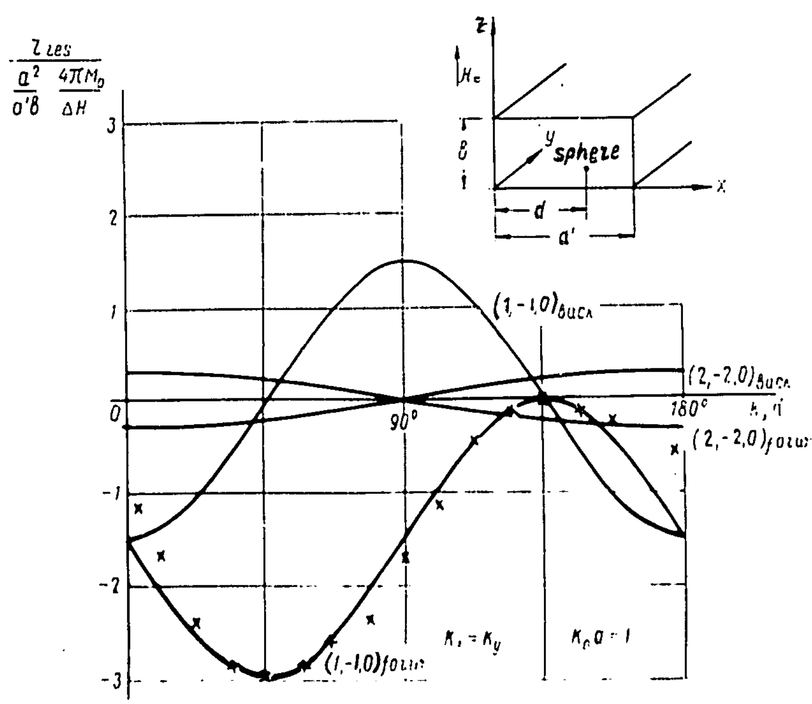

0,75
0,5
0,25
0
(1,-1,0)
(2,-1,0)
(3,-1,0)
(3,-1,0)foz
=1,373

Fig. 3


3
2
1
0
-1
-2
-3
90°
180°
sphere
d
a'
(1,-1,0)back
(2,-2,0)back
(2,-2,0)forw
(1,-1,0)forw

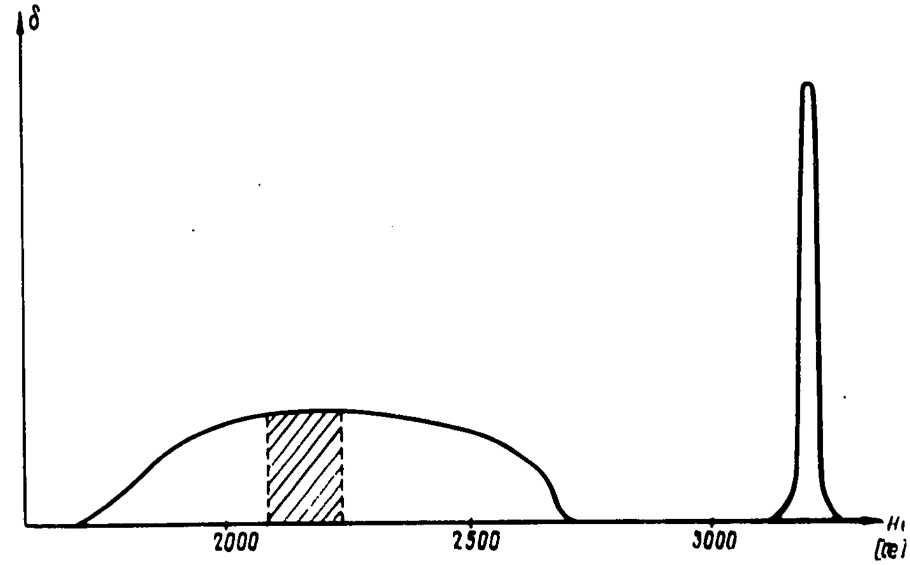
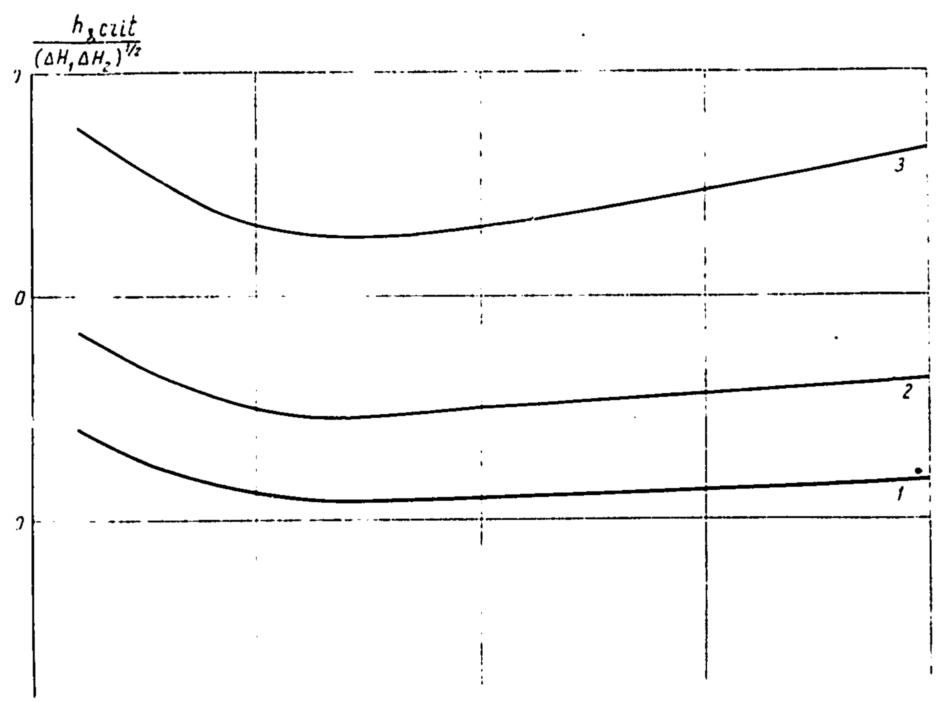


Fig. 6



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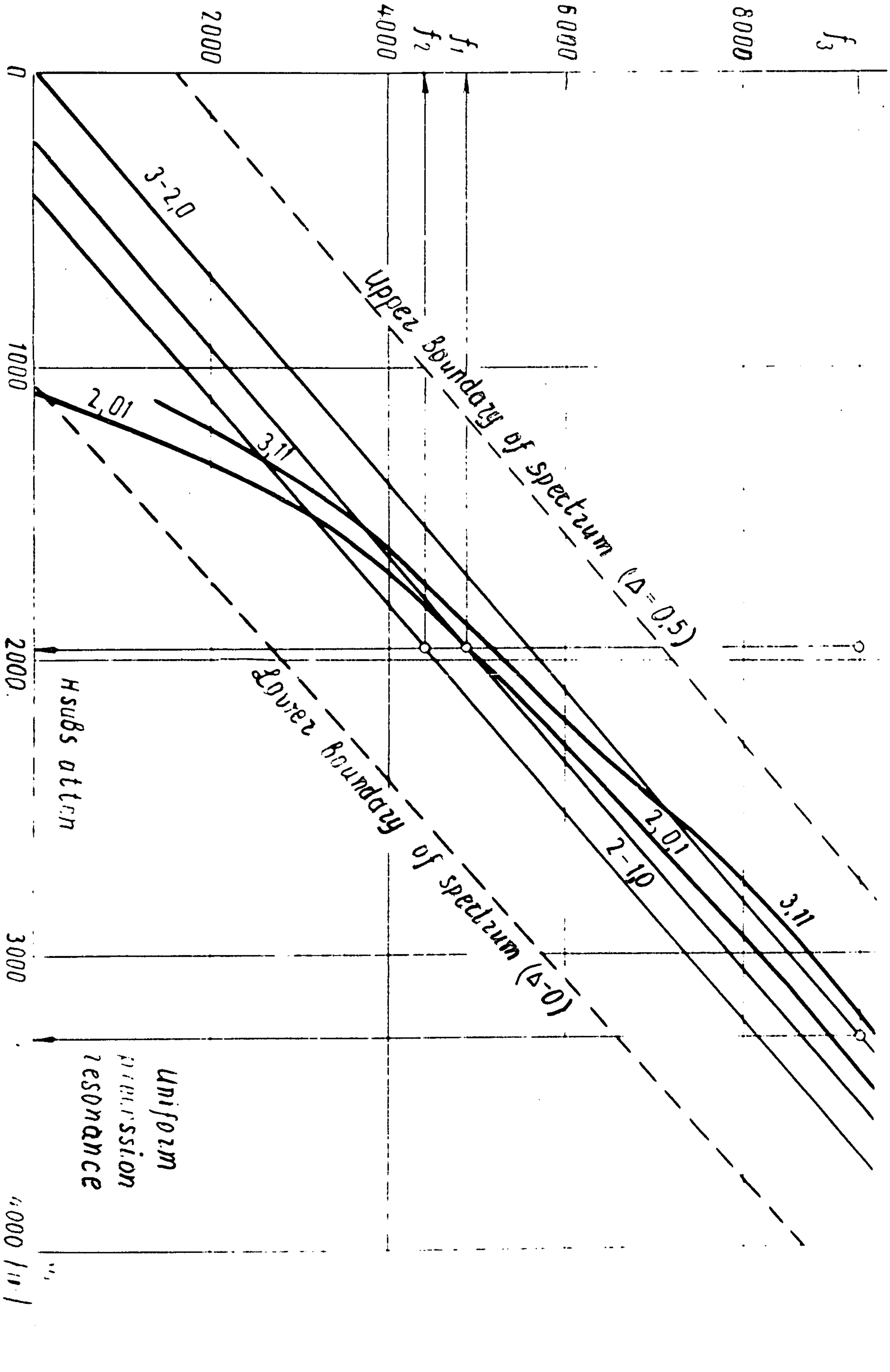
f_3
8000
6000
f_1
f_2
4000
2000
0
1000
2000
3000
3-2,0
2,01
3,11
2-1,0
Upper Boundary of Spectrum (Δ = 0,5)
Lower Boundary of Spectrum (Δ-0)
Uniform
resonance

Fig.5